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Indian Forest Service Main Examination

(2000-2023)

Civil Engineering

Paper-I

Topicwise Presentation

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Civil Engineering : Indian Forest Service Main Examination (Paper-I)

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Preface

Our country has a vast forest cover of near about 25% of geographical area and if man doesn't learn to treat trees with respect, man will become extinct; Death of forest is end of our life. Scientific management and judicial exploitation of forest becomes first task for sustainable development.

B. Singh (Ex. IES)

Engineer is one such profession which has an inbuilt word "Engineer – skillful arrangement" and hence IFS is one of the most talked about jobs among engineers to contribute their knowledge and skills for the arrangement and management for sustainable development

In order to reach to the estimable position of Divisional Forest Officer (DFO), one needs to take an arduous journey of Indian Forest Service Examination. Focused approach and strong determination are the pre-requisites for this journey. Besides this, a good book also comes in the list of essential commodity of this odyssey.

I feel extremely glad to launch the revised edition of such a book which will not only make Indian Forest Service Examination plain sailing, but also with 100% clarity in concepts.

MADE EASY team has prepared this book with utmost care and thorough study of all previous years' papers of Indian Forest Service Examination. The book aims to provide complete solution to all previous years' questions with accuracy.

On doing a detailed analysis of previous years' Indian Forest Service Examination question papers, it came to light that a good percentage of questions have been asked in Engineering Services, Indian Forest Services and State Services exams. Hence, this book is a one stop shop for all Indian Forest Service Examination, CSE, ESE and other competitive exam aspirants.

I would like to acknowledge efforts of entire MADE EASY team who worked day and night to solve previous years' papers in a limited time frame and I hope this book will prove to be an essential tool to succeed in competitive exams and my desire to serve student fraternity by providing best study material and quality guidance will get accomplished.

With Best Wishes

B. Singh

CMD, MADE EASY Group

Previous Years Solved Papers

Indian Forest Service Main Examination

Civil Engineering

Paper-I

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■■■■

1 Strength of Materials

1. Properties of Metal and Basic Concepts

1.1 The bars *AB*, *AC* and *AD* shown in indeterminate system (see figure below) are made of steel and have same cross-sectional area of 350 mm^2 and they together carry a load of 75 kN, applied at *A* as shown. There is no initial stress in bars before application of load. α = 30° and l = 3000 mm. Find force in each bar and vertical displacement of point A after load is applied. Take E = 205 kN/mm^2.

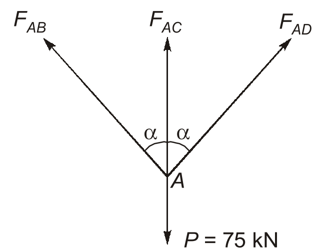
[15 marks : 2002]

Solution:

Let the force in *AB*, *AC* and *AD* are F_{AB}, F_{AC} and F_{AD}

$$\alpha = 30°$$

$$P = 75 \text{ kN}, \; L_{AC} = L = 3 \text{ m}$$



In equilibrium condition

$$\Sigma F_x = 0$$

$$\Rightarrow \quad F_{AB} \sin 30° - F_{AD} \sin 30° = 0$$

$$\Rightarrow \quad F_{AB} = F_{AD} \quad \text{...(i)}$$

$$\Sigma F_y = 0$$

$$\Rightarrow \quad F_{AB} \cos 30° + F_{AC} + F_{AD} \cos 30° = P$$

$$\Rightarrow \quad 2F_{AB} \times \frac{\sqrt{3}}{2} + F_{AC} = P$$

$$\Rightarrow \quad F_{AC} = \left(P - \sqrt{3}F_{AB}\right) \quad \text{...(ii)}$$

Total strain energy

$$U = V_{AB} + V_{AC} + V_{AD} = \frac{F_{AB}^2 L_{AB}}{2AE} + \frac{F_{AC}^2 L_{AC}}{2AE} + \frac{F_{AD}^2 L_{AD}}{2AE}$$

$$\Rightarrow \quad U = \left\{\frac{F_{AB}^2 \times 2L}{2AE \times \sqrt{3}} \times 2\right\} + \frac{\left(P - \sqrt{3}F_{AB}\right)^2 L}{2AE}$$

for minimum strain energy

$$\frac{\partial U}{\partial F_{AB}} = 0$$

$$\Rightarrow \quad \frac{L}{2AE}\left(\frac{4}{\sqrt{3}} \times 2F_A + 2\left(P - \sqrt{3}F_{AB}\right) \times -\sqrt{3}\right) = 0$$

$$\Rightarrow \quad F_{AB} = \frac{3P}{4 + 3\sqrt{3}} = \frac{3 \times 75}{4 + 3\sqrt{3}} = 24.47 \text{ kN}$$

$$F_{AD} = F_{AB} = 24.47 \text{ kN}$$

$$F_{AC} = P - \sqrt{3}F_A = 75 - \sqrt{3} \times 24.47 = 32.62 \text{ kN}$$

Vertical detection of point *A* (from Castigliano's Ist theorem)

$$\Delta V_A = \frac{\partial V}{\partial P} = \frac{\partial}{\partial P}\left\{\frac{2F_{AB}^2 \times 2L}{2AE \times \sqrt{3}} + \frac{\left(P - \sqrt{3}F_{AB}\right)^2 L}{2AE}\right\}$$

$$= \frac{L}{2AE}\left\{\frac{L}{\sqrt{3}} \times 2F_{AB} \times \frac{\partial F_{AB}}{\partial P} + 2\left(P - \sqrt{3}F_{AB}\right)L\left(1 - \sqrt{3}\frac{\partial F_{AB}}{\partial P}\right)\right\}$$

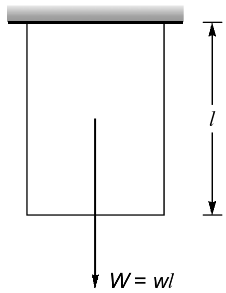
$$= \frac{1}{2AE}\left\{\frac{4L}{\sqrt{3}} \times 2F_{AB} \times \left(\frac{3}{4+3\sqrt{3}}\right) + 2\left(P - \sqrt{3}F_{AB}\right)L\left(1 - \sqrt{3} \times \frac{3}{4+3\sqrt{3}}\right)\right\}$$

$$= \frac{1}{(2 \times 350 \times 10^{-6} \times 205 \times 10^{6})}\left\{\frac{4 \times 3}{\sqrt{3}} \times 2 \times 24.47 \times \frac{3}{4+3\sqrt{3}}\right\}$$

$$+2 \times \left(75 - \sqrt{3} \times 24.47\right) \times 3\left(1 - \frac{3\sqrt{3}}{4+3\sqrt{3}}\right)$$

$$= \frac{195.72 \times 1}{2 \times 305 \times 205} = 1.36 \times 10^{-3}\text{m} = 1.36 \text{ mm}$$

1.2 Find elongation of a bar of uniform cross-section area 'A' and length 'l' under action of its own weight. The bar weight 'w' per unit length. E = Modulus of elasticity. See figure below:



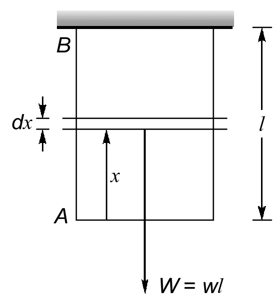
[10 marks : 2002]

Solution:

Take element of cross-sectional area 'A' with length 'dx' elongate under weight w'

$$w' = wx$$

Δl = Total elongation, dl = Elements elongation



$$dl = \frac{(wx)dx}{AE}$$

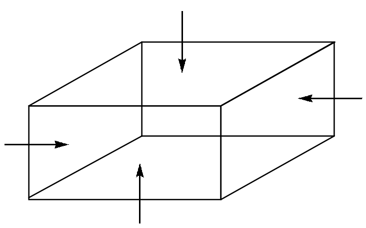
$$\int dl = \Delta l = \int_0^l \frac{(Wx)dx}{AE}$$

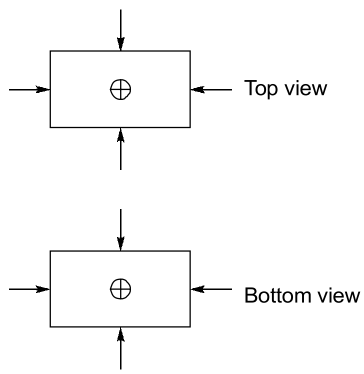
$$\Delta l = \frac{wl^2}{2AE} = \frac{Wl}{2AE}$$

1.3 A 50 mm cube is subjected to uniform pressure of 200 MPa. When the change in dimension between 2 parallel faces of cube is 0.025 mm. Determine change in volume of cube, μ = 0.25.

[8 marks : 2010]

Solution:





$$p = 200 \text{ MPa}$$

$$\Delta l = \left[-\frac{p}{E} + \mu \times \frac{p}{E} + \mu \frac{p}{E}\right] \times l$$

Given, $\Delta l = -0.025$ mm, $l = 50$ mm, p = 200 MPa

$$\frac{50 \times 200}{E} \times (2 \times 0.25 - 1) = -0.025$$

$$E = 2 \times 10^5 \text{ MPa}$$

$$B = \text{Bulk modulus} = \frac{E}{3(1-2\mu)}$$

$$B = \frac{2 \times 10^5}{3 \times (1 - 2 \times 0.25)} = 1.33 \times 10^5 \text{ MPa}$$

$$B = \frac{-p}{(\Delta V/V)}$$ (p should be insulated with sign)

$$1.33 \times 10^5 = \frac{-(-200)}{\Delta V/(50)^3}$$

$$\frac{\Delta V}{(50)^3} = 150 \times 10^{-5}$$

$$\mathbf{\Delta V = 187.5 \ mm^3}$$

Alternative

$$\Delta l_1 = 0.025 \text{ mm}$$

$$\frac{\Delta V}{V} = \epsilon_V = \epsilon_1 + \epsilon_2 + \epsilon_3$$

As it is cube so $\Delta l_1 = \Delta l_2 = \Delta l_3$ (Uniform pressure)

$$\epsilon_V = \left(\frac{0.025}{50}\right) \times 3 = 1.5 \times 10^{-3}$$

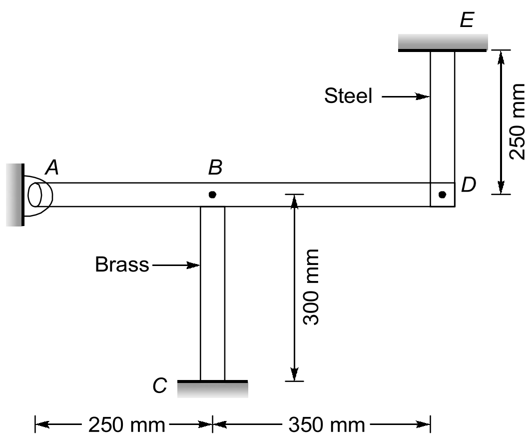
$$\frac{\Delta V}{V} = 1.5 \times 10^{-3}$$

$\Rightarrow$ $\Delta V = (1.5 \times 10^{-3})\, V$

$$\Delta V = 187.5 \text{ mm}^3$$

1.4 A rigid bar *AD* is pinned at *A* and attached to the bars *BC* and *ED* as shown in figure. The entire system in initially stress free and weights of all bars are negligible. The temperature of bar BC is lowered 25°C and of bar *ED* is raised 25°C. Neglecting any possibility of lateral buckling, find normal stress in bars BC and *ED*. For BC which is brass, assume E = 90 GPa, $\alpha = 20 \times 10^{-6}$/°C and for *ED*, which is steel take E = 200 GPa and $\alpha = 12 \times 10^{-6}$/°C. Cross-sectional area of *BC* is 500 mm² and of *ED* is 250 mm².

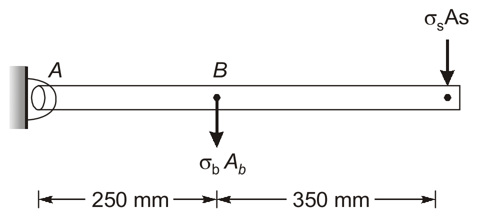
[10 marks : 2011]



Solution:

Let stress in brass = σ_b (Tensile)

Stress in steel = σ_s (Compressive) (Assume)



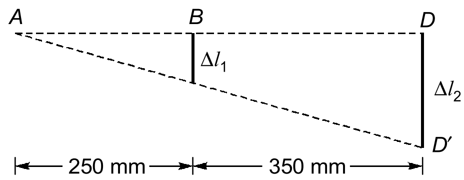
$$(\Sigma M)_A = 0 \quad \text{for static rotational equilibrium}$$

$$\sigma_b A_b \times 250 + \sigma_s A_s \times 600 = 0$$

$$\sigma_b \times 500 \times 250 + \sigma_s \times 250 \times 600 = 0$$

$$\sigma_b = (-\sigma_s)\,1.2 \qquad \text{...(i)}$$

Now as *ABD* bar is rigid so



$$\frac{\Delta l_1}{250} = \frac{\Delta l_2}{600}$$

$$\Delta l_2 = 2.4\Delta l_1$$

$$\Delta l_1 = 300 \times 20 \times 10^{-6} \times 25 - \frac{\sigma_b}{0.9 \times 10^5} \times 300$$

$$\Delta l_1 = 0.15 - 3.33 \times 10^{-3}\,\sigma_b \text{ (mm)}$$

All stress values are in N/mm^2

$$E = \text{in N/mm}^2 \text{ or MPa}$$

$$\Delta l_2 = l_s \alpha_s \Delta T - \frac{\sigma_s}{E_s} l_s$$

$$\Delta l_2 = 250 \times 12 \times 10^{-6} \times 25 - \frac{\sigma_s}{2 \times 10^5} \times 250$$

$$\Delta l_2 = 0.075 - 1.25 \times 10^{-3}\,\sigma_s$$

Now, $0.075 - 1.25 \times 10^{-3}\,\sigma_s = 2.4\,(0.15 - 3.33 \times 10^{-3}\,\sigma_b)$

$\Rightarrow$ $8 \times 10^{-3}\,\sigma_b - 1.25 \times 10^{-3}\,\sigma_s = 0.285$

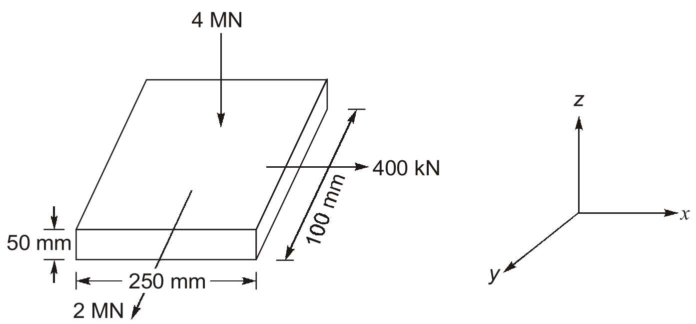
$$8\,\sigma_b - 1.25\,\sigma_s = 285 \qquad \text{...(ii)}$$

$$\sigma_b = 31.521 \text{ N/mm}^2 \quad \text{(Tensile)}$$

$$\sigma_s = -26.27 \text{ N/mm}^2$$

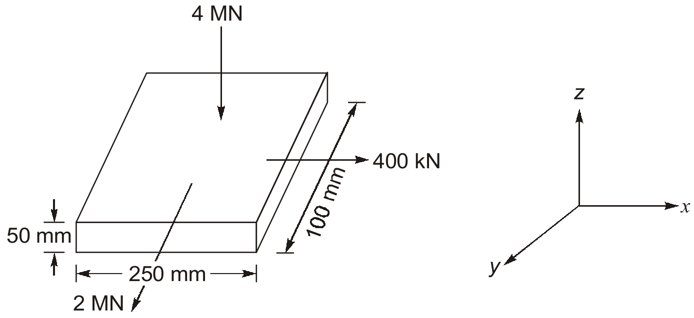
$$\sigma_s = 26.27 \text{ N/mm}^2 \text{ (Tensile)}$$

1.5 A metallic bar 250 mm× 100 mm × 50 mm is loaded as shown in below figure. Workout change in volume. What should be change that should be made in 4 MN load in order that there should be no change in the volume of the bar? Assume $E = 2 \times 10^5$ N/mm² and $\mu = 0.25$.



[15 marks : 2012]

Solution:



Stress in x-direction, $\sigma_x = \frac{2\times10^6}{250\times50} = 160$ N/mm^2 $\left\{\sigma = \frac{P}{A}\right\}$

Stress in y-direction, $\sigma_y = \frac{400\times10^3}{100\times50} = 80$ N/mm^2

Stress in z-direction, $\sigma_z = \frac{-4\times10^6}{100\times250} = -160$ N/mm^2

Strain in x-direction, $\in_x = \frac{\sigma_x}{E} - \frac{\mu(\sigma_y+\sigma_z)}{E}$

Strain in y-direction, $\in_y = \frac{\sigma_y}{E} - \frac{\mu(\sigma_x+\sigma_z)}{E}$

Strain in z-direction, $\in_z = \frac{\sigma_z}{E} - \frac{\mu(\sigma_x+\sigma_z)}{E}$

$\therefore$ Volumetric strain, $\in_V = \in_x + \in_y + \in_z = \frac{(\sigma_x+\sigma_y+\sigma_E)(1-2\mu)}{E}$

$\Rightarrow$ $\in_V = \frac{(160+80-160)}{2\times10^5} = 4\times10^{-4}$

$\therefore$ Change in volume, $\Delta v = \in_v \cdot V = 4\times10^{-4}\times(100\times250\times50)$

$= 500$ mm^2

Let the new stress in E-direction σ'_E so that volume charge is zero.

$$\Delta_V = \frac{(\sigma_x+\sigma_y+\sigma'_z)}{E}(1-2\mu)\cdot V = 0$$

$\Rightarrow$ $\sigma_x+\sigma_y+\sigma'_z = 0$

$\Rightarrow$ $\sigma'_z = -(\sigma_x+\sigma_y) = -(160+80) = -240$ N/mm^2

$P'_z = -240\times100\times250 = -6\times10^6$ N $= -6$ MN

$\therefore$ Change in load in z-direction, $\Delta P_z = -6-(-4) = -2$ MN

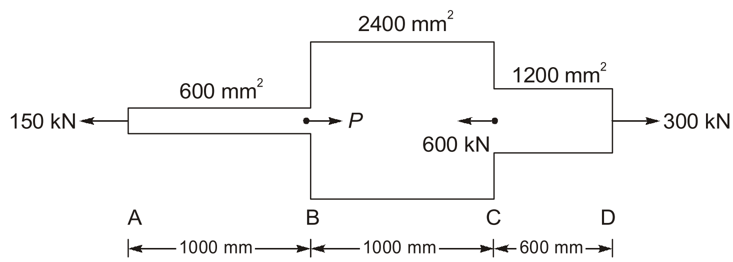
$= 2$ MN (compressive)

1.6 $E = 2 \times 10^5$ N/mm²

A member *ABCD* is subjected to concentrated loads as shown. Calculate

(i) Force *P* necessary for equilibrium

(ii) Total elongation of bar



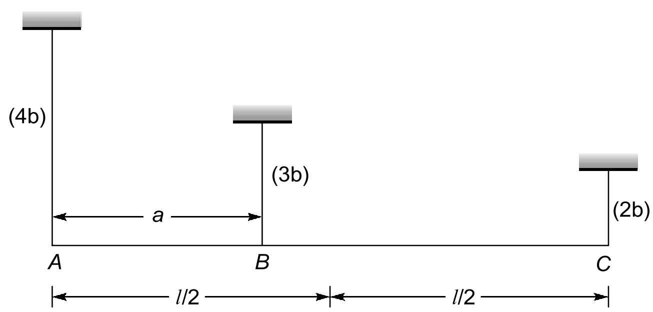
[8 marks : 2015]

Solution:

(i)
$$\Sigma F = 0$$
$$(P + 300) - (150 + 600) = 0$$
$$P = 450 \text{ kN}$$

(ii)
$$\Delta_{Total} = \Delta_{AB} + \Delta_{BC} + \Delta_{CD}$$
$$= \frac{150 \times 1000 \times 1000}{600 \times 2 \times 10^5} - \frac{300 \times 1000 \times 1000}{2400 \times 2 \times 10^5} + \frac{300 \times 600 \times 1000}{1200 \times 2 \times 10^5}$$
$$= 1.25 - 0.625 + 0.75$$
$$= 1.375 \text{ mm (elongation)}$$

1.7 Three steel bars *A*, *B* and *C* having same axial rigidity *AE* support a horizontal rigid beam *A*, *B*, *C* as shown in figure. Determine distance 'a' between bars *A* and *B*. In order that rigid beam will remain horizontal. When a load '*F*' is applied at its mid point. The value of length is given within parenthesis.



[8 marks : 2015]

Solution:

The beam is rigid and as per given condition/situation of beam to be remain horizontal following conditions must be satisfied.

1. Static equation
2. Axial elongation in all steel bars is equal and it takes place gradually.
3. There should be no net moment about any point.

From left, strings are named as 1, 2, and 3 respectively.

$$F_1 + F_2 + F_3 = F \quad \text{...(i)}$$
$$\frac{F_1(4b)}{AE} = \frac{F_2(3b)}{AE} = \frac{F_3(2b)}{AE} \quad \text{...(ii)}$$
$$4F_1 = 3F_2 = 2F_3 \quad \text{...(iii)}$$

$$\Rightarrow \quad F_1 + \frac{4F_1}{3} + \frac{4}{2}F_1 = F$$

$$\Rightarrow \quad \frac{13}{3}F_1 = F$$

$$\Rightarrow \quad F_1 = \frac{3}{13}F$$

$$\Rightarrow \quad F_2 = \frac{4}{13}F$$

$$\Rightarrow \quad F_3 = \frac{6F}{13}$$

Now, $(\Sigma M)_A = 0$ to prevent any rigid body rotation.

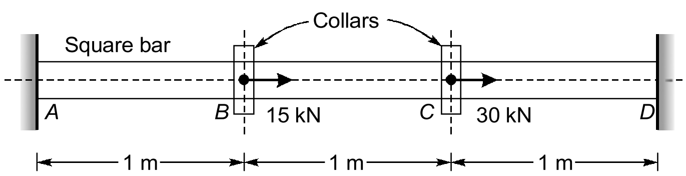
$$F_2(a) + F_3(l) = F\left(\frac{l}{2}\right)$$

$$\frac{4F}{13}a + \frac{6F}{13}l = \frac{Fl}{2}$$

$$\frac{4F}{13}a = Fl \times \frac{1}{26}$$

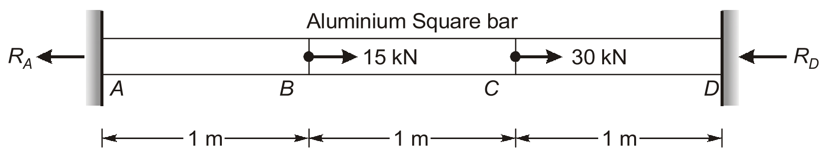
$$a = \frac{l}{8}$$

1.8 An aluminium square bar having the cross-section 50 mm × 50 mm and length 3 metres is fixed between two rigid supports as shown in the figure. Two loads, 15 kN and 30 kN are applied concentrically to the rod through collars as shown. Determine the stress developed at the right end of the bar. Young's modulus of aluminium is 70×10^9 N/m^2.



[10 marks : 2016]

Solution:

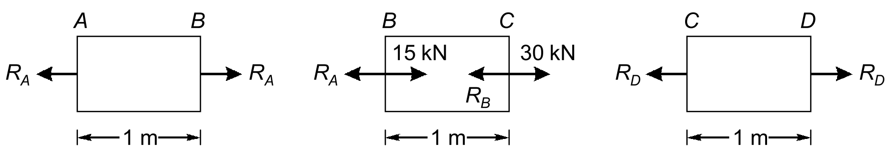


Given, Cross-section of aluminium bar = 50 mm × 50 mm

Young's no duels of aluminium, $E = 70 \times 10^9$ N/m^2

Let the reactions at A and D are R_A and R_D respectively.

Drawing free body diagrams individually;



Let there is tension in CD, let it be x, then

$$x = R_A - 15 = 30 - R_D$$

$$\Rightarrow \quad R_A + R_D = 45 \text{ kN} \qquad \ldots(i)$$

$$\Delta_{AB} = \frac{R_A \times 1000}{AE} \text{ (Elongation)} \quad (\because \text{ Bar is prismatic, } AE \text{ is constant})$$

$$\Delta_{BC} = \frac{x \times 1000}{AE} \text{ (Elongation)}$$

$$\Delta_{CD} = \frac{R_D \times 1000}{AE} \text{ (Compression)}$$

But A & D are fixed, $\Delta_{AB} + \Delta_{BC} + \Delta_{CD} = 0$

$$\frac{R_A \times 1000}{AE} + \frac{x \times 1000}{AE} - \frac{R_D \times 1000}{AE} = 0$$

$$R_A + x - R_D = 0$$

$$\Rightarrow \quad R_A + (R_A - 15) - R_D = 0$$

$$\Rightarrow \quad 2R_A - 15 - R_D = 0$$

$$\Rightarrow \quad 2R_A - R_D = 15 \quad \text{...(ii)}$$

Solving equation (i) and (ii), we get $R_A = 20$ kN

$$R_D = 25 \text{ kN}$$

$\therefore$ Stress developed on right end of bar = Stress at fixed end D

$$\sigma_0 = \frac{-R_D}{A} = \left(\frac{-25000}{50 \times 50}\right) = -10 \text{ N/mm}^2 \text{ (Compression)}$$

1.9 If two pieces of materials 'A' and 'B' have the same bulk modulus, but the value of Modulus of Elasticity for 'B' is 1% greater than that for 'A', find the value of Modulus of Rigidity for the material 'B' in terms of Modulus of Elasticity and Modulus of Rigidity for material 'A'.

[8 marks : 2017]

Solution:

Let E_A, K_A, G_A and E_B, K_B, G_B be the modulus of elasticity, bulk modulus and modulus of rigidity of materials A and B respectively.

Given, $K_A = K_B$ and $E_B = 1.01\, E_A$

We know, $$E = \frac{9KG}{3K + G}$$

or, $$3KE + EG = 9\, KG$$

or, $$3K(3G - E) = EG$$

from where, we get $$K = \frac{EG}{3(3G - E)}$$

Hence, $$\frac{E_A G_A}{3(3G_A - E_A)} = \frac{E_B G_B}{3(3G_B - E_B)}$$

$$\therefore \quad E_A G_A(3G_B - E_B) = E_B G_B(3G_A - E_A)$$

or, $$3E_A G_A G_B - E_A G_A E_B = 3G_A G_B E_B - E_A E_B G_B$$

or, $$G_B(3E_A G_A - 3G_A E_B + E_A E_B) = E_A G_A E_B$$

$$\therefore \quad G_B = \frac{E_A G_A E_B}{3E_A G_A - 3G_A E_B + E_A E_B}$$

$$\Rightarrow \quad G_B = \frac{1.01 E_A G_A E_A}{3E_A G_A - 3 \times 1.01 E_A G_A + 1.01 E_A \times E_A}$$

or, $$G_B = \frac{1.01 E_A G_A}{3G_A - 3.03 G_A + 1.01 E_A} = \frac{101 E_A G_A}{101 E_A - 3G_A}$$

1.10 A leaf spring of semi-elliptical type has 11 plates each 9 cm wide and 1.5 cm thick. The length of spring is 1.5 m. The plates are made of steel having a proof stress (bending) of 650 MN/m². To what radius should the plates be bent initially? From what height can a load of 600 N fall on to centre of the spring, if maximum stress is to be one-half of the proof stress? Take E = 200 GN/m².

[8 marks : 2018]

Solution:

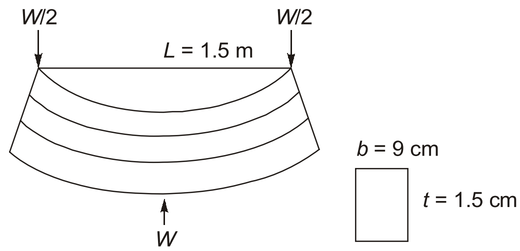
$$f_y = 650 \text{ N/mm}^2$$

$$\because \quad \frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$I = \frac{bt^3}{12}$$

$$y_{max} = \frac{t}{2}$$

$$E = 200 \text{ GPa} = 200 \times 10^6 \text{ N/mm}^2$$



$$\Rightarrow \quad R = \frac{E \cdot y_{max}}{\sigma_y} = \frac{200 \times 10^3 \times \left(\frac{15}{2}\right)}{650} \text{mm}$$

$$= 2307.69 \text{ mm} = 2.307 \text{ m}$$

$\therefore$ Initial radius, R = 2.307 m

Let the load P = 600 N falls from x height

$$\sigma_{max} = \frac{1}{2} \times \sigma_y = \frac{650}{2} = 325 \text{ N/mm}^2$$

$$P = 600 \text{ N}$$

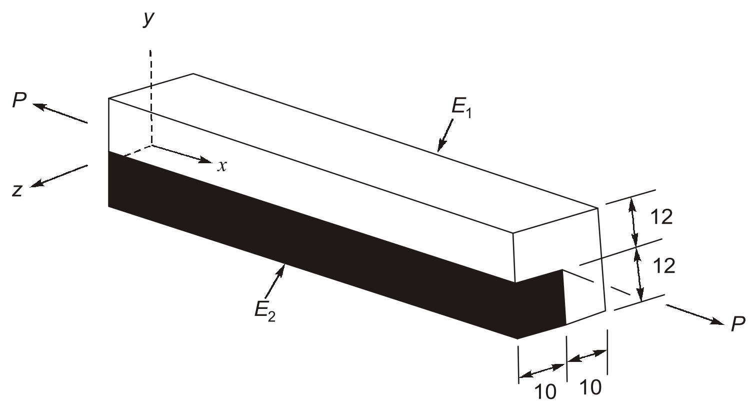
From energy conservation $$Px = \frac{\sigma_y^2}{2E} \times V$$

$$\Rightarrow \quad 600x = \frac{(325 \times 10^6)^2}{2 \times 200 \times 10^9} \times 11 \times 9 \times 1.5 \times 10^{-9} \times 1.5$$

$$\Rightarrow \quad x = 9.80 \text{ m}$$

$\therefore$ Height of fall = 9.80 m

1.11 A composite bar of rectangular cross-section 20 mm × 24 mm is loaded in tension by a force P = 1 kN as shown in fig. below. The shaded part of the bar is made of a material where Young's modulus is E_2 = 210 GPa. The remaining part is made of a material with E_1 = 105 GPa. If the bar is to deflect in the x-direction, determine the stresses in each material and the location of the loading axis relative to the centre of the bar:



[10 Marks : 2019]

Solution:

Given, $E_1 = 105$ GPa; $E_2 = 210$ GPa

$A_1 = 12 \times 20 + 12 \times 10 = 360 \text{ mm}^2$

$A_2 = 12 \times 10 = 120 \text{ mm}^2$

There is deformation in only x-direction.

∵ In composite bar, deformation will be same

∴ for uniform deformation.

$$(\varepsilon_x)_1 = (\varepsilon_x)_2$$

$$\frac{(\sigma_x)_1}{E_1} = \frac{(\sigma_x)_2}{E_2}$$

$$\frac{(\sigma_x)_1}{(\sigma_x)_2} = \frac{E_1}{E_2} = \frac{105}{210} = \frac{1}{2} = 0.5 \qquad ...(1)$$

For composite bar, $P = P_1 + P_2$

$$1000 = (\sigma_x)_1 A_1 + (\sigma_x)_2 A_2$$

$$= 0.5(\sigma_x)_2 A_1 + (\sigma_x)_2 A_2$$

$$= 0.5\,(\sigma_x)_2 \times 360 + (\sigma_x)_2 \times 120$$

$$1000 = 300\,(\sigma_x)_2$$

$$(\sigma_x)_2 = \frac{1000}{300} = 3.33 \text{ MPa}$$

from eq. (1) $(\sigma_x)_1 = 0.5 \times 3.33 = 1.665$ MPa

Let force is acting at a point Q having y-coordinate e_y and z-coordinate $(-e_z)$

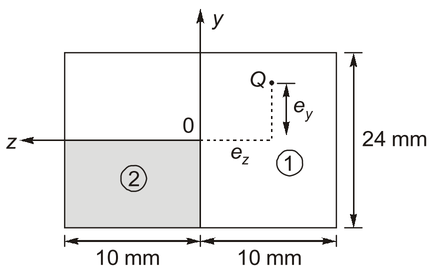
Centroid of section (1)

$$\bar{y}_1 = \frac{120 \times 6 + 240 \times 0}{360} = 2 \text{ mm}$$

$$\bar{z}_1 = \frac{120 \times 5 + 240 \times (-5)}{360} = \frac{-5}{3} \text{mm}$$

Centroid of section (2) $\bar{y}_2 = -6$ mm

$\bar{z}_2 = 5$ mm



By moment equilibrium equation

$$\Sigma(M_0)_y = 0$$

$$1000 \times e_y = P_1\bar{y}_1 + P_2\bar{y}_2$$

$$1000 \times e_y = (\sigma_x)_1 A_1 \cdot \bar{y}_1 + (\sigma_x)_2 \cdot A_2\bar{y}_2$$

$$1000 \times e_y = 1.665 \times 360 \times 2 + 3.333 \times 120 \times (-6)$$

$$e_y = -1.20 \text{ mm}$$

Now, $\Sigma(M_0)_z = 0$

$$1000 \times e_z = P_1\bar{z}_1 + P_2 \cdot \bar{z}_2$$

$$1000 \times e_z = (\sigma_x)_1 A_1\bar{z}_1 + (\sigma_x)_2 A_2 \cdot \bar{z}_2$$

$$1000 \times e_z = 1.665 \times 360 \times \left(-\frac{5}{3}\right) + (3.333 \times 120 \times 5)$$

Hence, location of loading axis relative to the centre of bar is (–1.2 mm, 1 mm)

1.12 In a tension test, a steel rod of gauge length 255 mm and diameter 32 mm was used. The rod during the test was stretched 0.108 mm under a pull of 65 kN. In a torsion test, the same rod was twisted 0.018 radian over a length of 255 mm at the torque of 500 × 10^3 N-mm. Determine the modulus of elasticity, modulus of rigidity, Poisson's ratio and bulk modulus.

[8 Marks : 2022]

Solution:

Given :

Length of rod, $L = 255$ mm

Diameter of rod, $d = 32$ mm

Pull applied, $P = 65 \times 10^3$ N

Elongation in rod, $\Delta = 0.108$ mm

As we know, that when a rod of uniform cross-section is subjected to an axial load,

then, $$\Delta = \frac{Pl}{AE} \qquad \left[\because A = \frac{\pi}{4}d^2\right]$$

where Δ is axial deformation/elongation and E is modulus of elasticity

Putting values, we get

$$0.108 = \frac{65 \times 10^3 \times 255}{\frac{\pi}{4} \times 32^2 \times E}$$

$$\Rightarrow \quad E = 190827.05 \text{ N/mm}^2$$

Now, when it is subjected to a torsion of 500 × 10^3 N-mm, angle of twist θ is 0.018 radian.

By torsion formula,

$$\frac{T}{I_P} = \frac{G\theta}{L}$$

where, T = Torsion applied; I_P = Polar moment of inertia of rod = $\frac{\pi d^4}{32}$

G = Modulus of rigidity; θ = Angle of twist; L = Length of rod

Putting values, we get

$$\frac{500 \times 10^3}{\frac{\pi}{32} \times 32^4} = \frac{G \times 0.018}{255}$$

$$\Rightarrow \quad G = 68807.83 \text{ N/mm}^2$$

As we know $\quad E = 2G(1 + \mu)$ where μ is Poisson's ratio

Putting values, we get

$$190827.05 = 2 \times 68807.83(1 + \mu)$$

$$\Rightarrow \quad \mu = 0.387$$

Also, $\quad E = 3K(1 - 2\mu)$ where K is bulk modulus

Putting values, we get

$$190827.05 = 3 \times K \times (1 - 2 \times 0.387)$$

$$\Rightarrow \quad K = 281455.83 \text{ N/mm}^2$$

Hence,

Modulus of elasticity, E = 190827.05 N/mm^2 = 1.9 × 10^5 N/mm^2

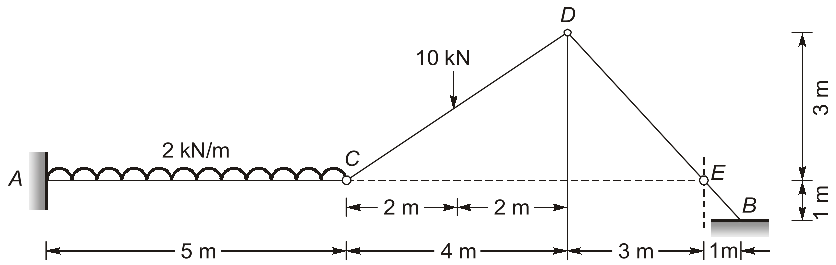
Modulus of rigidity, G = 68807.83 N/mm^2 ≃ 0.688 × 10^5 N/mm^2

Bulk modulus of elasticity, K = 281455.83 N/mm^2 ≃ 2.8 × 10^5 N/mm^2

Poisson's ratio, μ = 0.387

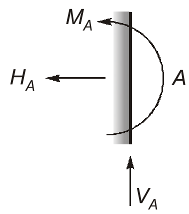
2. Shear Force and Bending Moment

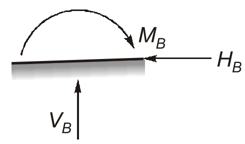
2.1 A structural frame is loaded as shown in figure. It has 3 hinges at C, D and E. C and E being at same level. Calculate reaction at A and B.



[20 marks : 2006]

Solution:





$$r_e = 3 + 3 = 6$$
$$m = 4$$
$$r' = 3 \text{ (3 internal hinge)}$$
$$j = 5$$
$$D_{Se} = 3m + r_e - 3j - r'$$
$$= 3 \times 4 + 6 - 3 \times 5 - 3$$
$$= 0 \text{ (Statically determinate)}$$
$$V_A + V_B = 20 \quad \text{...(i)}$$
$$H_A + H_B = 0 \quad \text{...(ii)}$$
$$(\Sigma M)_A = 0$$
$$M_A + 13V_B - M_B - H_B \times 1 - 25 - 10 \times 7 = 0$$
$$M_A + 13V_B - M_B - H_B = 95 \quad \text{...(iii)}$$

$M_C = 0$ (Left) $\quad 5V_A - M_A = 25 \quad$ ...(iv)

$M_D = 0$ (Right) $\quad 4V_B = 4H_B + M_B \quad$ ...(v)

$M_E = 0$ (Right) $\quad V_B = H_B + M_B \quad$ ...(vi)

From eq. (v) and (vi) $\quad M_B = 0$

$$V_B = 4H_B$$
$$V_A = 20 - H_B$$
$$100 - 5H_B - 25 = M_A$$
$$M_A = 75 - 5H_B$$
$$75 - 5H_B + 13H_B - 0 - H_B = 95$$
$$7H_B = 20$$

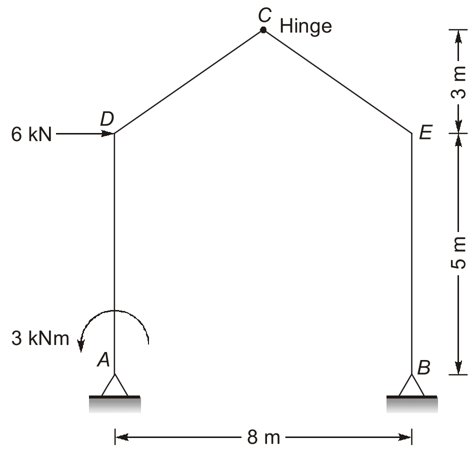
$\Rightarrow$ $$H_B = \frac{20}{7} = 2.86 \text{ kN}$$

$\Rightarrow$ $$H_A = -2.86 \text{ kN}$$

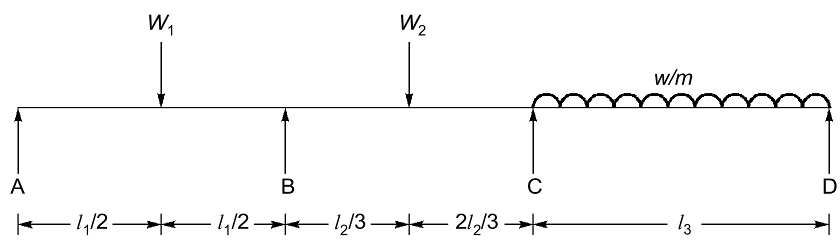
$$V_B = 2.86 \text{ kN}$$
$$V_A = 17.14 \text{ kN}$$
$$5 \times 16.8 - M_A = 25$$

$\Rightarrow$ $$M_A = 60.7 \text{ kNm}$$

2.2 (i) Find support reactions at A and B of structure shown in figure. The structure has an internal hinge at C.



(ii) Draw the free body diagram for spans AB, BC and CD and show thrust acting on support A, B, C and D.



[4 + 4 = 8 marks : 2015]

Solution:

(i)

$$H_A + H_B = 6 \quad \text{...(i)}$$

$$V_A + V_B = 0 \quad \text{...(ii)}$$

$$(\Sigma M)_A = 0$$

$$8V_B + 3 = 6 \times 5$$

$$\mathbf{V_B = 3.375\ kN} \uparrow$$

$$\mathbf{V_A = 3.375\ kN} \downarrow$$

$$M_C = 0$$

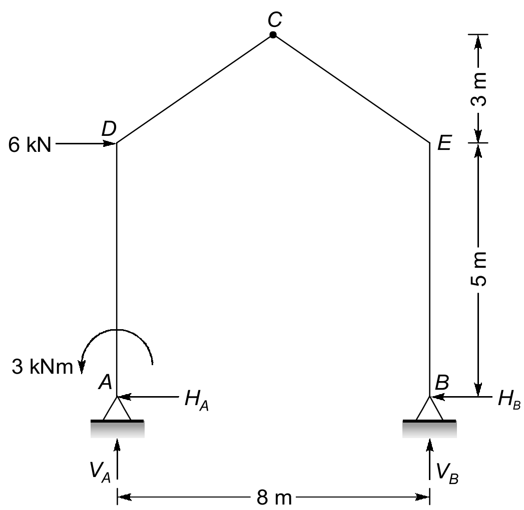
Taking moment about C from right side

$$8H_B = 4V_B$$

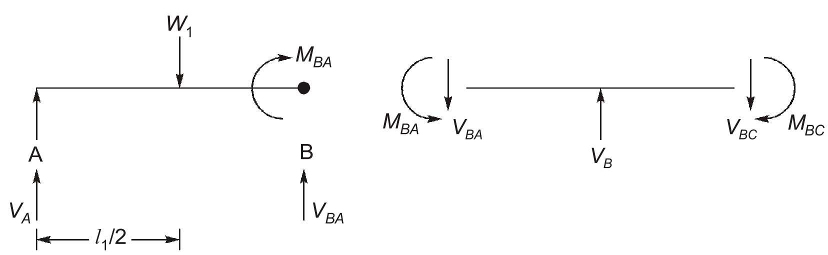
$$\mathbf{H_B = 1.6875\ kN} \leftarrow$$

$$H_A = 6 - H_B$$

$$\mathbf{H_A = 4.3125\ kN} \rightarrow$$

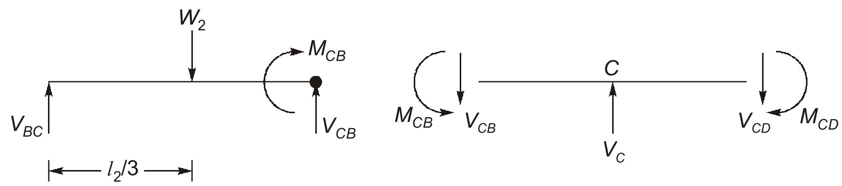


(ii)



$$V_A + V_{BA} = W_1 \quad \text{...(i)}$$

$$V_{BA} + V_{BC} = V_B \quad \text{...(ii) (Joint support } B\text{)}$$



$$V_{CB} + V_{CD} = V_C \quad \text{...(iii)}$$

$$M_{BA} + M_{BC} = 0 \quad \text{...(iv)}$$

$$M_{CB} + M_{CD} = 0 \quad \text{...(v)}$$

$$V_{BC} + V_{CB} = W_2 \quad \text{...(vi)}$$

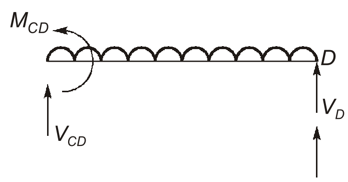
$$V_{CD} + V_D = Wl_3 \quad \text{...(vii)}$$

A and *D* are simple pinned discontinuous supports hence

$$M_A, M_D = 0$$

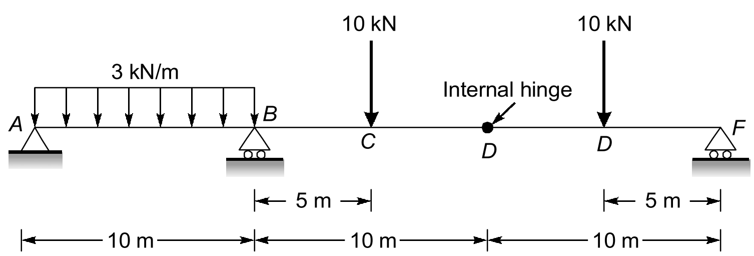
While *B* and *C* are continuous supports

$$M_B, M_C \neq 0$$



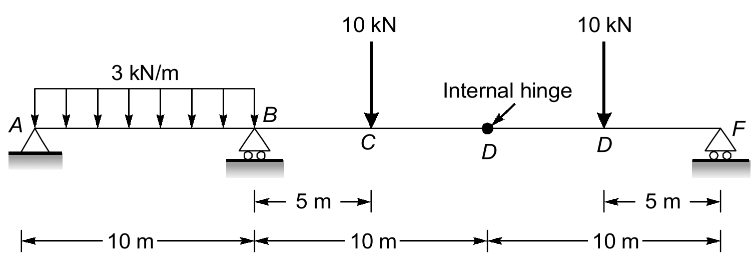
Equation of equilibrium of joint resists accompanied by static equilibrium.

2.3 Draw the bending moment and shear force diagrams of the following beam as shown in the figure. The beam has an internal hinge at *D*.



[15 marks : 2016]

Solution:



Shear force diagram:

Let V_A, V_B and V_F be the support reactions

$$\therefore \quad M_O = 0$$

Taking moment from RHS,

$$V_F \times 10 = 10 \times 5$$

$$\Rightarrow \quad V_F = 5 \text{ kN } (\uparrow)$$

Similarly taking moment from LHS,

$$V_A \times 20 + V_B \times 10 = 3 \times 10 \times 15 + 10 \times 5$$

$$\Rightarrow \quad 20\,V_A + 10\,V_B = 500 \text{ kN}$$

$$\Rightarrow \quad 2V_A + V_B = 50 \quad \text{...(i)}$$

For equilibrium of beam, $\Sigma F_y = 0$

$$\Rightarrow \quad V_A + V_B + V_F - 30 - 10 - 10 = 0$$

$$\Rightarrow \quad V_A + V_B + V_F = 50$$

$$\Rightarrow \quad V_A + V_B = 45 \text{ kN} \quad \text{...(ii)}$$

Solving eq. (i) and (ii), we get

$$V_A = 5 \text{ kN } (\uparrow),\ V_B = 40 \text{ kN } (\uparrow)$$

For portion AB; $\quad SF_x = V_A - 3x = 5 - 3x$

At $x = 0$, $\quad (SF)_A = 5 \text{ kN } (\uparrow)$

At $x = 10$ m, $\quad (SF)_B = 5 - 3 \times 10 = -25$ kN

For $(SF)_x = 0$

$$\Rightarrow \quad 5 = 3x$$

$$\Rightarrow \quad x = 1.67 \text{ m}$$

For portion BD;

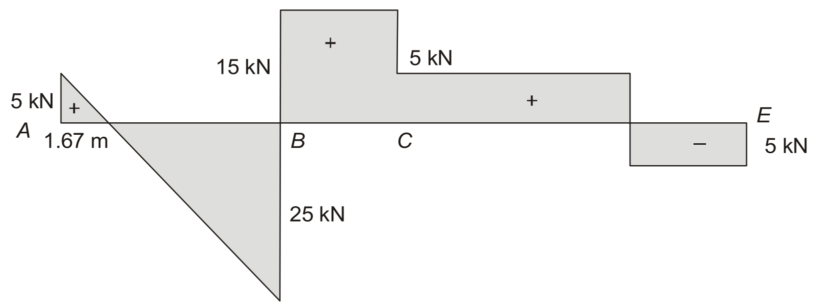
$$(SF)_D = V_A - 3 \times 10 + V_B - 10 = 45 - 40 = 5 \text{ kN}$$

$$(SF)_B = -25 \text{ kN} + 40 = 15 \text{ kN}$$

For portion DF;

$$(SF)_E = (SF)_D - 10 = (5 - 10) = -5 \text{ kN}$$

$$(SF)_F = -5 \text{ kN}$$



For portion AB; $(0 < x \le 10 \text{ m})$ $\quad M_x = V_A x - \dfrac{3x^2}{2} = 5x - 1.5x^2$

At $x = 0$, $M_A = 0$

and $\quad x = B = 10 \text{ m} = 50 - 1.5 \times 100 = -100$ kNm

For maximum value, $\quad \dfrac{dM_x}{dx} = -5 - 3x$

$$x = \frac{5}{3} = 1.67 \text{ m}$$

$$M_x = 0 = x\,(5 - 1.5\,x)$$

$$x = \frac{5}{1.5} = 3.33 \text{ m}$$

$$M_{max} = 5 \times 1.67 - 1.5 \times (1.67 \times 1.67) = 4.167 \text{ kNm}$$

For portion BC, $(0 < x < 5 \text{ m})$ $\quad M_x = 5(x + 10) + 40x - 30\,(5 + x)$

At $x = 0$ $\quad M_B = 50 - 30 \times 5 = -100$ kNm

At $x = 5$ $\quad M_C = 5 \times 15 + 40 \times 5 - 30 \times 10 = 75 + 200 - 300 = -25$ kNm

For portion CD; $(0 < x < 5)$ $\quad M_x = 5\,(x + 15) + 40\,(x + 5) - 30\,(x + 10) - 10 \times x$

At $x = 0$ $\quad M_C = -25$ kNm

At $x = 5$ $\quad M_D = 100 + 400 - 450 - 10 \times 5 = 0$

From RHS of the beam;

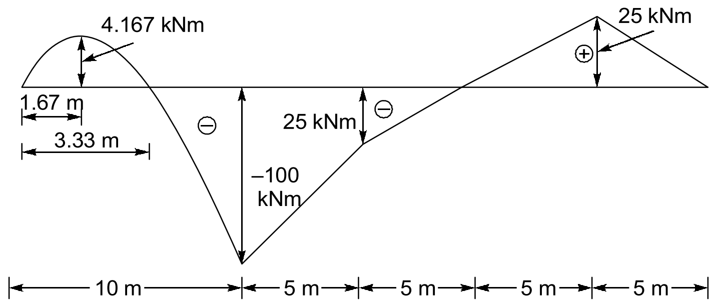
Portion FD; $0 < x < 5$ $\qquad M_x = 5x$

At $x = 0$, $\qquad M_F = 0$

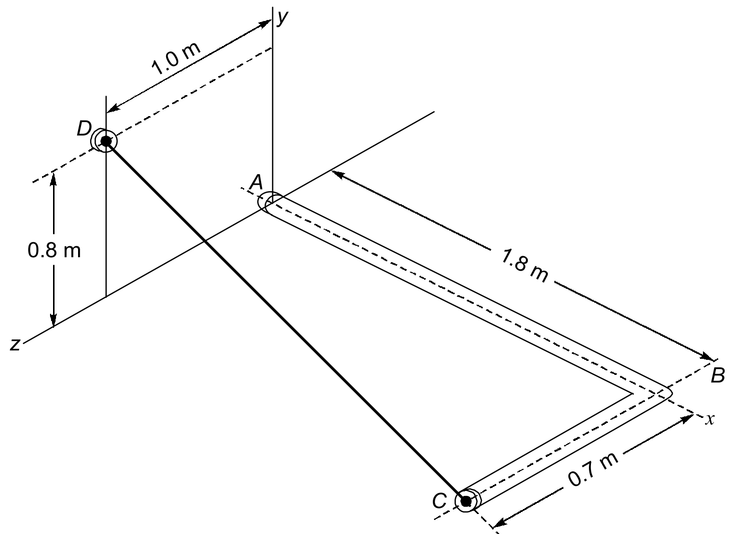
At $x = 5$ m, $\qquad M_F = 25$ kNm

Portion ED; $0 < x < 5$ $\qquad M_x = 5(x + 5) - 10x$

At $x = 0$ $\qquad M_D = 50 - 50 = 0$



2.4 A right-angled rigid pipe is fixed to the wall at A and is additionally supported through the cable CD as shown in the figure. Determine the magnitudes of the moments about the x, y and z axes, if the tensile force applied to the cable is 3 kN.

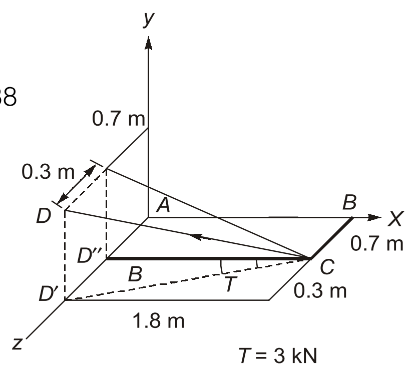


[8 marks : 2016]

Solution:

In $\Delta DD'C$

$$\tan\alpha = \frac{DD'}{CD'} = \frac{0.80}{\sqrt{0.3^2 + 1.8^2}} = 0.438$$



$\Rightarrow$ $$\alpha = 23.67°$$

In $D'D''C$

$$\tan\beta = \frac{D'D''}{CD''} = \frac{0.3}{1.8}$$

$\Rightarrow$ $$\beta = 20.56°$$

$$\vec{T_C} = (T\cos\alpha)\cos\beta(-\hat{i}) + (T\sin\alpha)\hat{j} + (T\cos\alpha)\sin\beta\hat{k}$$

$$= 3\cos 23.67° \times \cos 20.56°\ (-\hat{i}) + 3\sin 23.67°\ \hat{j} + 3\cos 23.67°$$

$$\times \sin 20.56\ \hat{k}$$

$$= 2.57(-\hat{i}) + 1.204(\hat{k}) + 0.965\hat{k}$$

$$\overrightarrow{A_C} = 1.8\hat{i} + 0.7\hat{k}$$

$$\therefore \quad \vec{M} = \overrightarrow{A_C} \times \overrightarrow{T_C} = (1.8\hat{i} + 0.7\hat{k}) \times \left\{2.57(-\hat{i}) + 1.204\hat{j} + 0.965\hat{k}\right\}$$

$$= 1.8 \times 1.204\hat{k} + 1.8 \times 0.965(-\hat{j}) + 0.7 \times 2.57(-\hat{j}) + 0.7 \times 1.204(-\hat{i})$$

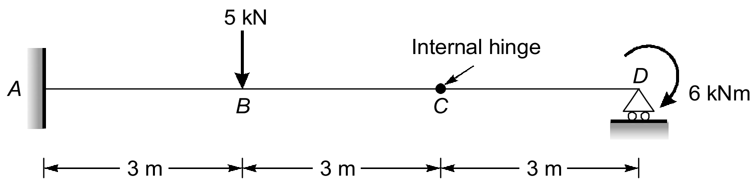
$$= 0.843(-\hat{i}) - 3.536(\hat{j}) + 2.167\hat{k}$$

$$M_x = 0.8431(-\hat{i})\ \text{kNm}$$

$$M_y = -3.536(\hat{j})\ \text{kNm}$$

$$M_z = 2.167(\hat{k})\ \text{kNm}$$

2.5 Find the support reactions of the beam shown in the figure. The beam has an internal hinge at *C*.



[8 marks : 2016]

Solution:

Let V_A and V_D be the support reactions at *A* & *D* respectively and M_A be the bending moment at *A*.

We know, $\Sigma F_V = 0$

$\Rightarrow \quad V_A + V_D = 5$ kN

Taking moment about hinge

$M_C = 0$

$\Rightarrow \quad V_D \times 3 = 6$

$\Rightarrow \quad V_D = 2$ kN (↑)

$\therefore \quad V_A = (5 - 2) = 3$ kN(↑)

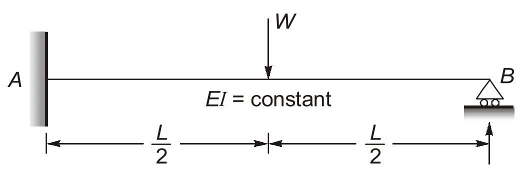
Taking moment about *C* from LHS;

$V_A \times 6 + M_A = 5 \times 3$

$\Rightarrow \quad 3 \times 6 + M_A = 15$

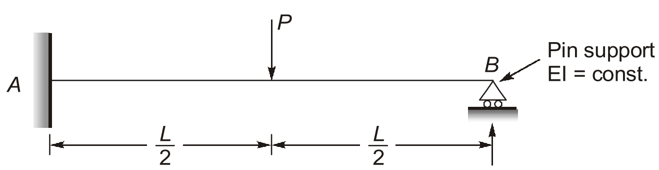
$\Rightarrow \quad M_A = -3$ kNm

2.6 Using column analogy method, determine the bending moment at fixed end in the propped cantilever beam as shown in the figure below:

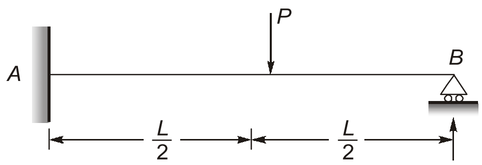


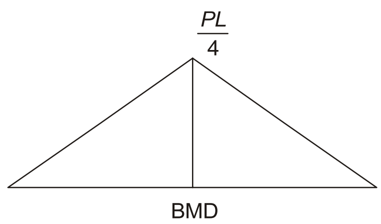
[8 marks : 2018]

Solution:

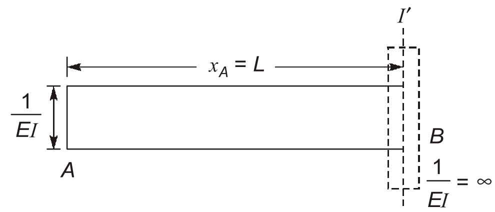


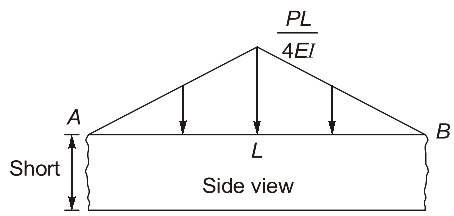
Corresponding statically determinate beam (simply supported)





Analogous column





Pressure at base of analogous column

$$P = \frac{F}{A} \pm \frac{M'}{I'}x$$

$$F = \frac{1}{2} \times L \times \frac{PL}{4EI} = \frac{PL^2}{8EI}$$

$$A = \frac{1}{EI} \times L + \infty = \infty$$

$$I' = \frac{1}{EI} \times \frac{L^3}{3} = \frac{L^3}{3EI}$$

$$M' = \frac{PL^2}{8EI} \times \frac{L}{2} = \frac{PL^3}{16EI}$$

$$x_A = L, x_B = 0$$

$$P_A = \frac{F}{A} - \frac{M'}{I'}x_A = \frac{PL^2}{8EI} \times \frac{1}{\infty} + \frac{PL^3/16EI}{L^3/3EI} \times L = \frac{3PL}{16}$$

$$P_B = \frac{F}{A} - \frac{M'}{I'}x_B = \frac{PL^2}{8EI} \times \frac{1}{\infty} - \frac{PL^3/16EI}{L^3/3EI} \times 0 = 0$$

$\therefore$

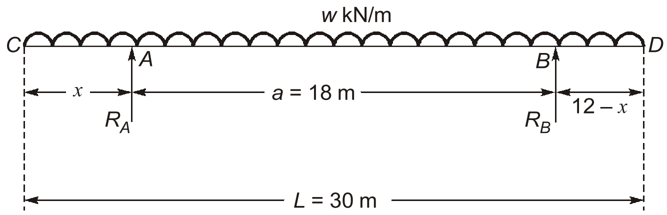
$$M_A = 0 + \frac{3PL}{16} = \frac{3PL}{16}$$

$$M_B = 0 + 0 = 0$$

2.7 A girder 30 m long carrying a uniformly distributed load of 'W' kN/m throughout is to be supported on two piers 18 m apart so that the greatest Bending Moment shall be as small as possible. Find the distance of the piers from the ends of the girder and the maximum Bending Moment.

[15 marks : 2020]

Solution:



The girder CD is 30 m long

Let the two pier 18 m apart are at A and B.

Let x = Distance of pier A from C (in m)

Then, distance of pier B from end D

$$= 30 - (18 + x) = (12 - x) \text{ m}$$

Calculation of reactions at piers:

$$\Sigma F_V = 0$$

$$R_A + R_B = 30\,w \quad \text{...(i)}$$

$$\Sigma M_A = 0$$

$$\frac{wx^2}{2} + R_B \times 18 = (30 - x) \times w \times \left(\frac{30 - x}{2}\right)$$

$$\Rightarrow \quad 0.5wx^2 + 18R_B = \frac{(30 - x)^2}{2} w$$

$$\Rightarrow \quad 0.5wx^2 + 18R_B = \left(\frac{900 + x^2 - 60x}{2}\right) w$$

$$\Rightarrow \quad 18R_B = \left(\frac{900 - 60x}{2}\right) w$$

$$\Rightarrow \quad R_B = \left(\frac{900 - 60x}{36}\right) w$$

$$= \left(25 - \frac{5}{3}x\right) w$$

∴ From eq. (i)

$$R_A = 30w - R_B = 30w - \left(25 - \frac{5}{3}x\right) w$$

$$= 5w + \frac{5}{3}xw = 5w\left(1 + \frac{x}{3}\right)$$

In the present case of overhanging beam, the maximum negative BM will be at either of the two piers and the maximum positive BM will be in the span AB. If the BM on the beam is as small as possible, then the length of the overhanging portion should be so adjusted that the maximum negative BM at the piers is equal to the maximum positive BM in the span AB.

The BM will be maximum in the span AB at a point where SF is zero.

Let BM is maximum (or SF is zero) at a section in AB at a distance 'l' m from C.

∴ SF at this section $= -w \times l + R_A$

or,

$$-w \times l + 5w\left(1 + \frac{x}{3}\right) = 0$$

$$\Rightarrow \quad l = 5\left(1 + \frac{x}{3}\right) \quad \text{...(ii)}$$

Now, bending moment at piers A $= -\dfrac{wx^2}{2}$

And BM at a distance 'l' from C

$$= -w \times l \times \frac{l}{2} + R_A(l-x)$$

$$= -\frac{w}{2}\left[5\left(1+\frac{x}{3}\right)\right]^2 + 5w\left(1+\frac{x}{3}\right)\left(5+\frac{5x}{3}-x\right)$$

$$= -\frac{w}{2}(25)\left(1+\frac{x}{3}\right)^2 + 5w\left(1+\frac{x}{3}\right)\left(5+\frac{2}{3}x\right)$$

For the condition that the BM shall be as small as possible the hogging moment at the piers A and maximum sagging span moment in AB should be numerically equal

i.e., $$\frac{wx^2}{2} = 5w\left(1+\frac{x}{3}\right)\left(5+\frac{2}{3}x\right) - \frac{25}{2}w\left(1+\frac{x}{3}\right)^2$$

$$\Rightarrow \quad \frac{x^2}{2} = 5\left(1+\frac{x}{3}\right)\left(5+\frac{2}{3}x\right) - 12.5\left(1+\frac{x}{3}\right)^2$$

$$\Rightarrow \quad \frac{x^2}{2} = 5\left(5+\frac{2x}{3}+\frac{5x}{3}+\frac{2x^2}{9}\right) - 12.5\left(1+\frac{x^2}{9}+\frac{2x}{3}\right)$$

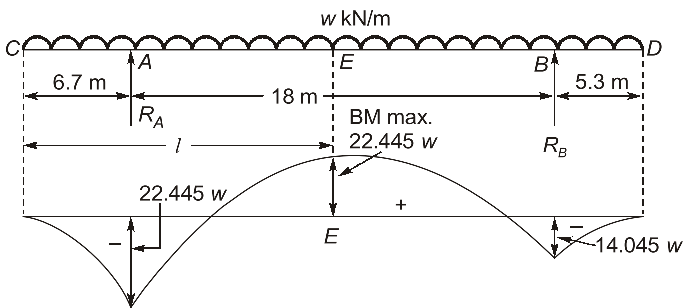
$$\Rightarrow \quad \frac{x^2}{2} = 25+\frac{35x}{3}+\frac{10x^2}{9} - 12.5 - \frac{12.5x^2}{9} - \frac{25x}{3}$$

$$\Rightarrow \quad \frac{x^2}{2} = 12.5+\frac{10x}{3} - \frac{2.5x^2}{9}$$

$$\Rightarrow \quad \frac{7}{9}x^2 - \frac{10x}{3} - 12.5 = 0$$

After solving, $x = 6688 \text{ m} \simeq 6.7 \text{ m}$

∴ Loading on the girder is as shown.



Bending Moment Diagram

Reaction at piers R_A and R_B:

$$R_A = 5w\left(1+\frac{6.7}{3}\right) = 16.17w$$

∴ $$R_B = \left(25-\frac{5}{3}\times 6.7\right)w = 13.83w$$

∴ BM at C = 0

$$\text{BM at A} = -w \times 6.7 \times \frac{(6.7)}{2} = -22.445w$$

BM at E (i.e., at a distance, $l = 5\left(1+\frac{x}{3}\right) = 5\left(1+\frac{x}{3}\right) = 5\left(1+\frac{6.7}{3}\right) = 16.17$ m)

$$BM_E = (BM_{max\,+ve}) = -w \times 16.17 \times \frac{16.17}{2} + R_A \times (16.17 - 6.7)$$

$$= -w \times 16.17 \times \frac{16.17}{2} + 16.17w(16.17 - 6.7)$$

$$\simeq 22.445\,w$$

BM at B, $$BM_B = -w \times 5.3 \times \frac{5.3}{2} = -14.045w$$

The bending moment between C and A; between A and B; and between B and D varies according to parabolic law as shown above.

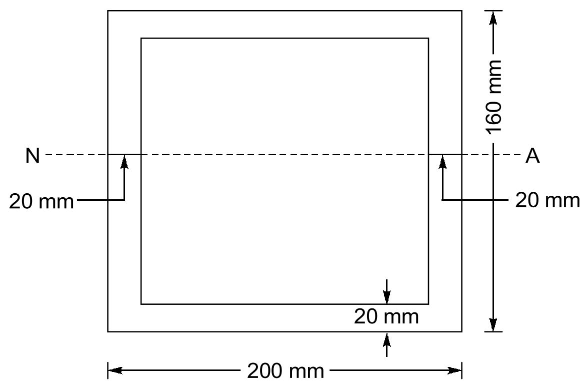
3. Bending Stress and Shear Stress

3.1 A simply supported hollow rectangular beam of outside width 200 mm, outside depth 160 mm and material thickness 20 mm is subjected to UDL of 10 kN/m for entire span of 10 m. Find maximum shear stress induced in beam.

[10 marks : 2000]

Solution:

$$q = \frac{V(A\bar{y})}{Ib}$$



Shear stress will be maximum at Neutral Axis because $(A\bar{y})$ will be maximum at NA coupled with minimum value of web thickness.

Also $$V_{max} = \frac{10 \times 10}{2} = 50 \text{ kN}$$

$$I = \frac{200 \times 160^3}{12} - \frac{(200 - 2\times 20)\times(160 - 2\times 20)^3}{12}$$

$$I = \frac{13568 \times 10^4}{3} \text{ mm}^4$$

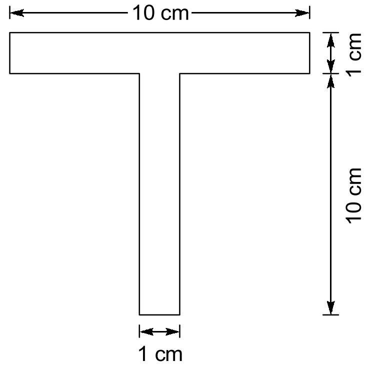
b at Neutral axis = 2 × 20 = 40 mm

$$A\bar{y} = 2\times[60\times 20\times 30] + 200\times 20\times 70 = 352000 \text{ mm}^3$$

$$b = 40 \text{ mm}$$

$$q = \frac{V(A\bar{y})}{Ib} = \frac{50\times 1000\times 352000}{\frac{13568\times 10^4}{3}\times 40} = 9.73 \text{ N/mm}^2$$

3.2 A T-beam (x-section shown in figure) is simply supported at ends over span of 7 m. It is subjected to UDL of 600 kgf/m (6 kN/m). 1°/C its own weight.
Calculate maximum shear stress in flange and at ends.
I_{xx} = 235.42 cm^4
centroid at 3.25 cm from Top.



[10 marks : 2006]

Solution:

$$\text{UDL} = 6 \text{ kN/m}$$

$$\text{Shear force at ends} = \frac{6 \times 7}{2} = 21 \text{ kN}$$

$$\text{Shear stress } (\tau) = \frac{V(A\bar{y})}{Ib}$$

I = Moment of area
A = Area above the section where τ is to be determined
$\bar{y}$ = distance of centroid of A from N.A.
b = width of section $\quad I = 2354200 \text{ mm}^4$

Maximum shear stress will be at Neutral axis because $A\bar{y}$ is maximum at NA while 'b' is minimum

$$A\bar{y} = 100 \times 10 \times 27.5 + 22.5 \times 10 \times \frac{22.5}{2}$$

$$A\bar{y} = 30031.25 \text{ mm}^3$$

$$b = 10 \text{ mm}$$

$$\tau_{max} = \frac{VA\bar{y}}{Ib} = \frac{21 \times 1000 \times 30031.25}{2354200 \times 10} = 26.79 \text{ N/mm}^2$$

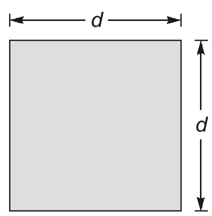
Shear stress will be zero on whole of section at mid span.

3.3 Compare bending strength of three beams one having a square cross-section, a rectangular section (depth is twice the width) and a circular cross-section; all the three beams having same weight and having a cross-sectional area of 95000 sq. mm each.

[10 marks : 2015]

Solution:

Same weight having same cross-sectional area implies that they have same material and equal value of maximum bending stress.



$$A = 95000 \text{ mm}^2$$

Case-1: Square $\quad d^2 = 95000$

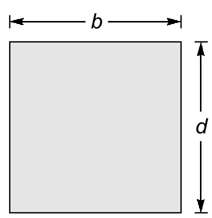
$\Rightarrow \quad d = 308.22 \text{ mm}$

$$\text{Bending strength} = \sigma_{max} \times \text{Section modulus } (z)$$

$$z = \left.\frac{bd^2}{6} = \frac{d^3}{6}\right|_{square}$$

$$z = \left(\frac{308.22}{6}\right)^3$$

$$(z)_1 = 4.88 \times 10^6 \text{ mm}^3$$



Case-2: Rectangle $\quad bd = 95000$

$$2b^2 = 95000$$

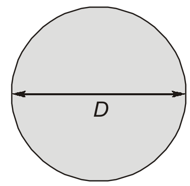
$\Rightarrow \quad b = 217.94 \text{ mm}$

$$d = 435.9 \text{ mm}$$

$$(z)_2 = \frac{bd^2}{6} = \frac{2}{3}b^3 = \frac{2}{3} \times 217.94^3$$

$$(z)_2 = 6.90 \times 10^6 \text{ mm}^3$$

Case-3: Circular



$$\frac{\pi D^2}{4} = 95000 \qquad \pi = \frac{22}{7}$$

$\Rightarrow$
$$D = 347.72 \text{ mm}$$

$$(z)_3 = \frac{\frac{\pi D^4}{64}}{(D/2)} = \frac{\pi D^3}{32} = \frac{22}{7} \times \frac{317.72^3}{32}$$

$$(z)_3 = 4.13 \times 10^6 \text{ mm}^3$$

$$(z)_2 > (z_1) > (z)_3$$

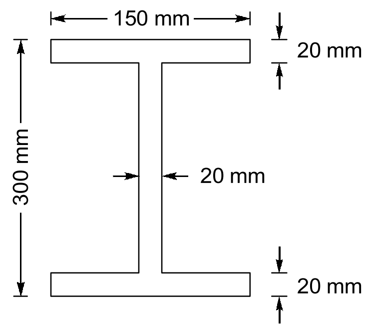
$$\text{Bending strength} = M$$

$$M_2 > M_1 > M_3$$

3.4 A steel beam having cross-section of an 'I' with overall depth 300 mm and flange width 150 mm is simply supported at both ends. The thickness of the flange, as well as the web is 20 mm for each. The beam needs to carry a concentrated load of 50 kN at its mid span. If the permissible bending stress is to be limited to 120 N/mm^2, determine

(i) the maximum possible length of the beam,

(ii) the depth of an equivalent rectangular section, with the width fixed to be 100 mm. Also, determine the percentage increase in weight of the beam as compared to the beam with '*I*' section.



[10 marks : 2016]

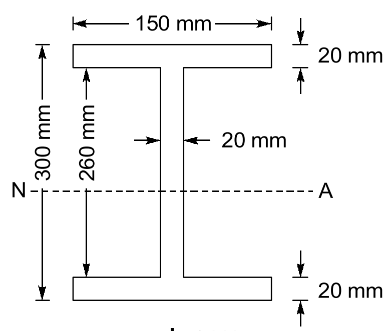
Solution:

Given, Maximum concentrated load,

$$W = 50 \text{ kN}$$

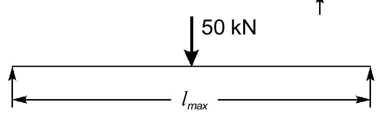
Permissible bending stress, $\sigma_{B,max} = 120 \text{ N/mm}^2$

(i) Let l_{max} be maximum possible length of beam.



$\therefore$
$$M_{max} = \frac{Wl_{max}}{4} = \frac{50\, l_{max}}{4}$$

$$= 12.5\, l_{max} \text{ kNm}$$



We know,
$$\frac{M_{max}}{I} = \frac{\sigma_{B_1 max.}}{y_{max}}$$

$$I = \frac{20 \times (260)^3}{12} + 150 \times 20(130+10)^2 + 150 \times 20 \times (140)^2$$

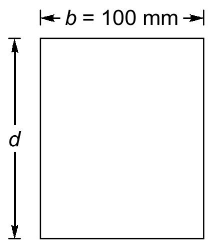
$$= 146.893 \times 10^6 \text{ mm}^4$$

$$\Rightarrow \quad \frac{12.5 l_{max} \times 10^6}{146.893 \times 10^6} = \frac{120}{150}$$

$$\Rightarrow \quad l_{max} = 9.4 \text{ metre}$$

(ii)

$$I = \frac{bd^3}{12} = \frac{100d^3}{12}$$



Let d be depth of equivalent section (rectangular)

$$\therefore \quad \frac{M}{I} = \frac{\sigma_{B_1 max.}}{y}$$

$$\Rightarrow \quad \frac{12.5 \times 9.4 \times 10^6 \times 12}{100 \times d^3} = \frac{120 \times 2}{d}$$

$$\Rightarrow \quad d = 242.4 \text{ mm}$$

We know, weight of beam = $\rho g \times$ Area of cross-section $\times$ length

$$\Rightarrow \quad W \propto A$$

Let weight of equivalent rectangular beam, $W_s \propto A_r$

$$\Rightarrow \quad A_r = bd = 242.384 \times 100 = 24238.4 \text{ mm}^2$$

Weight of I-section beam α A_I

$$A_I = 150 \times 20 \times 2 + 260 \times 20$$

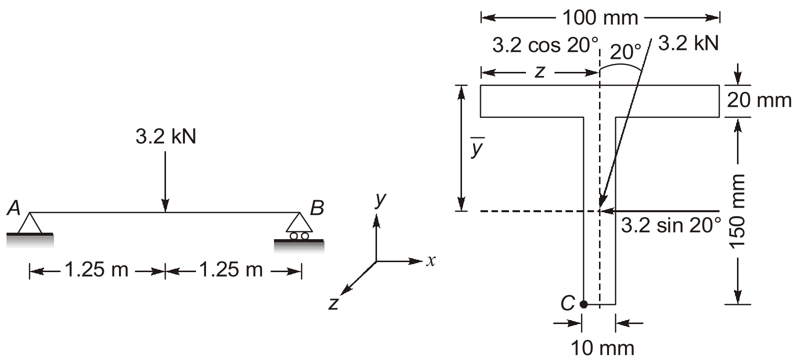
$$\Rightarrow \quad A_I = 11200 \text{ mm}^2$$

Hence, $$\% \text{ increase} = \left(\frac{24238.4 - 11200}{11200}\right) \times 100 = 116.41\%$$

3.5 A simply supported beam of T-section (flange = 100 mm × 20 mm and Web = 150 mm × 10 mm) is 2.5 m in length. It carries a load of 3.2 kN inclined at 20° to the vertical and passing through the centroid of the section. Determine the maximum tensile stress induced in the section.

[8 Marks : 2023]

Solution:



Note: Here, it is assumed that load acts at mid span of beam as it is not given in the problem.

So, maximum bending moment in z-direction, $M_z = \dfrac{P\cos 20° \times L}{4}$

$$= \frac{3.2 \times \cos 20° \times 2.5}{4} = 1.88 \text{ kN-m}$$

So, maximum bending moment in y-direction, $M_y = \dfrac{P\sin 20° \times L}{4}$

$$= \frac{3.2 \times \sin 20° \times 2.5}{4} = 0.68 \text{ kN-m}$$

Now, $$\bar{y} = \frac{100 \times 20 \times 10 + 150 \times 10 \times 95}{100 \times 20 + 150 \times 10} = 46.43 \text{ mm}$$

So, $$I_{zz} = \frac{100 \times 20^3}{12} + 100 \times 20 \times (46.43 - 10)^2 + \frac{10 \times 150^3}{12} + 10 \times 150 \times (95 - 46.43)^2$$

$$= 9.07 \times 10^6 \text{ mm}^4$$

$$I_{y\text{-}y} = \frac{20 \times 100^3}{12} + \frac{150 \times 10^3}{12}$$

$$= 1.68 \times 10^6 \text{ mm}^4$$

Now, maximum tensile stress will be at point C and its value is given as

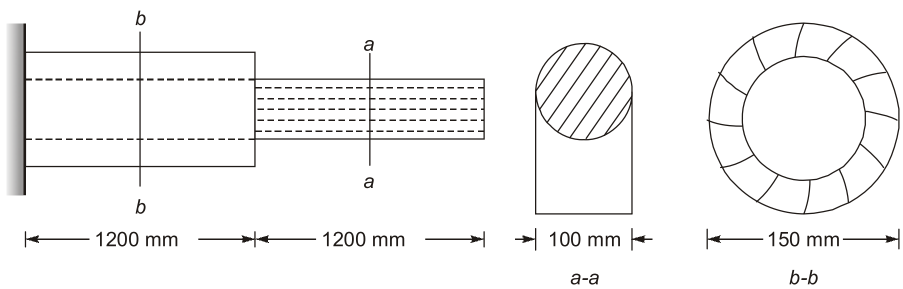
$$\sigma = \frac{M_z}{I_z} \times y + \frac{M_y}{I_y} \times z$$

$$= \frac{1.88 \times 10^6}{9.07 \times 10^6} \times (170 - 46.43) + \frac{0.68 \times 10^6}{1.68 \times 10^6} \times \frac{10}{2}$$

$$= 27.64 \text{ N/mm}^2$$

4. Torsion

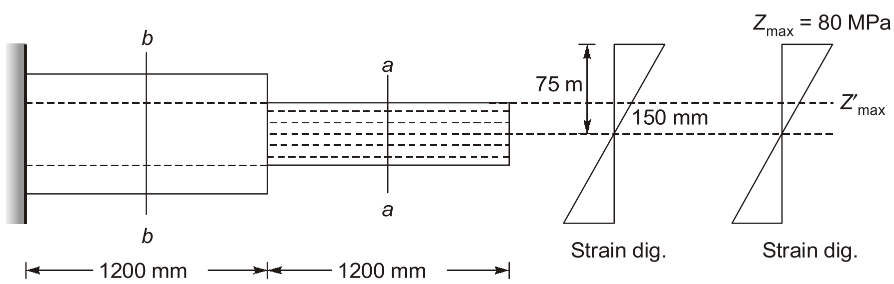
4.1 A shaft is made up of partly solid section and partly hollow section as shown as figure.



What is maximum torque that can be transmitted when maximum shear stress is 80 MPa, with modulus of rigidity = 80 GPa? What is maximum free end rotation?

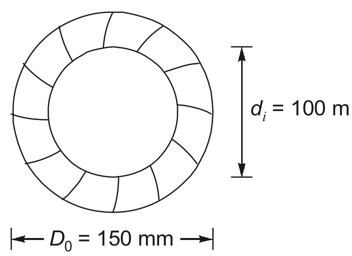
[12 marks : 2010]

Solution:



Let Maximum torque = T

For hollow section



$$\tau = \frac{T\, r_{max}}{I_P} = \frac{T\left(\frac{150}{2}\right) \times 10^6}{\pi \times \left(\frac{150^4 - 100^4}{32}\right)}$$

$$\Rightarrow \quad 80 = 1.8797\,T$$

$$\Rightarrow \quad T = 42.56 \text{ kNm} \quad \text{...(1)}$$

For sold section

$$Z_{max} = Z_{max} \times \frac{50}{75} = 80 \times \frac{50}{75} = 53.33 \text{ MPa}$$

$$Z_{max} = \frac{T\, r_{max}}{I_f} = \frac{T \times 10^6}{\pi \times \left(\frac{100^3}{16}\right)} = 5.091T$$

$$53.33 = 5.091\,T$$

$$T = 10.47 \text{ kNm} \quad \text{...(2)}$$

$$\Rightarrow \quad \text{Maximum torque} = \text{Minimum of (1) and (2)}$$

$$= 10.47 \text{ kNm}$$

$$Q_{max} = \frac{TL_1}{GI_{P_1}} + \frac{TL_2}{GI_{P_2}} = \frac{10.47 \times 10^6}{80 \times 1000} \times \left[\frac{1200 \times 32}{\pi(150^4 - 100^4)} + \frac{1200 \times 32}{\pi \times 100^4}\right]$$

$$= 19.93 \times 10^{-3} \text{ rad}$$

4.2 Find the diameter of a solid cylindrical shaft subjected to 100 rpm and transmitting 350 kW power, where shear stress not to exceed 90 N/mm^2. What percentage saving in weight would be obtained is shaft is replaced by hollow are, whose internal diameter equal to 0.65 of external diameter, the length, the material and maximum shear stress being the same.

[15 marks : 2012]

Solution:

$N = 100$ rpm, $P = 350$ kW, $P = T \times \omega$

$$350 \times 10^3 = T \times 2 \times \frac{22}{7} \times \frac{100}{60}$$

$$T = 33.41 \times 10^3 \text{ Nm}$$

$$\frac{\tau}{r} = \frac{T}{J}$$

$$J = \text{Polar moment of inertia} = \frac{\pi D^4}{32}$$

$$\tau = 90 \text{ MPa (maximum)}$$

So

$$\tau = \frac{16T}{\pi D^3}$$

$$D^3 = \frac{16 \times 33.41 \times 10^6 \times 7}{22 \times 90}$$

$$D_{min} = 123.63 \text{ mm}$$

Hollow shaft,

$$D_i = 0.65\text{ D}$$

$$J = \frac{\pi\left(D^4 - (0.65D)^4\right)}{32} = \frac{0.8215\pi D^4}{32}$$

$$\frac{\tau}{r} = \frac{T}{J}$$

$$\frac{90}{D/2} = \frac{33.41 \times 10^6}{\frac{0.8215\pi D^4}{32}}$$

$$\frac{90}{D} = \frac{33.41\times 10^6}{\frac{0.8215\pi D^4}{16}}$$

$$D^3 = \frac{33.41\times 10^6 \times 16\times 7}{90\times 0.8215\times 22}$$

$$D = 132.01 \text{ mm}$$

$$D_i = 0.65\, D = 85.81 \text{ mm}$$

$$(A_2) \text{ Area of hollow shaft} = k\times (D^2 - D_i^2) = k\times 10063.284 \qquad k = \frac{\pi}{4}$$

$$(A_1) \text{ Area of solid shaft} = kD^2 = k\times 15284.377$$

$$\%\text{ saving} = \frac{|A_2 - A_1|}{A_1}\times 100 = 34.16\%$$

4.3 The internal diameter of a steel shaft is 65% of external diameter. The shaft is to transmit 3600 kW at 210 rpm. If maximum allowable stress in shaft material is 50 N/mm², calculate diameter of shaft. Find also the maximum twist of shaft when it is stressed to maximum permissible value length of shaft is 3.8 m. Take G = 80 MPa.

[15 marks : 2015]

Solution:

Let external diameter = D mm, Internal diameter = 0.65 D mm, Power = Torque × ω

$$3600\times 10^3 = \text{Torque}\times \frac{210\times 2\times \frac{22}{7}}{60}$$

$$\text{Torque} = 163636.36 \text{ Nm}$$

$$\frac{\tau}{r} = \frac{T}{(\text{Polar moment of inertia})}$$

$$\frac{\tau_{max}}{D/2} = \frac{T}{\frac{\pi}{32}\left(D^4 - (0.65D)^4\right)}$$

$$\Rightarrow \quad \frac{2\times 50}{D} = \frac{163636.36\times 1000}{\frac{22}{7\times 32}D^4\left(1-0.65^4\right)}$$

$$\Rightarrow \quad D = 272.71 \text{ mm (External dia.)}$$

$$\text{Internal dia.} = 0.65D = 0.65\times 272.71 = 177.26 \text{ mm}$$

$$\frac{\tau}{r} = \frac{T}{J} = \frac{G\theta}{L}$$

$$G = 80 \text{ MPa}, L = 3.8 \text{ m}$$

$$\frac{50}{\left(\frac{272.71}{2}\right)} = \frac{80000\times \theta}{3.8\times 1000}$$

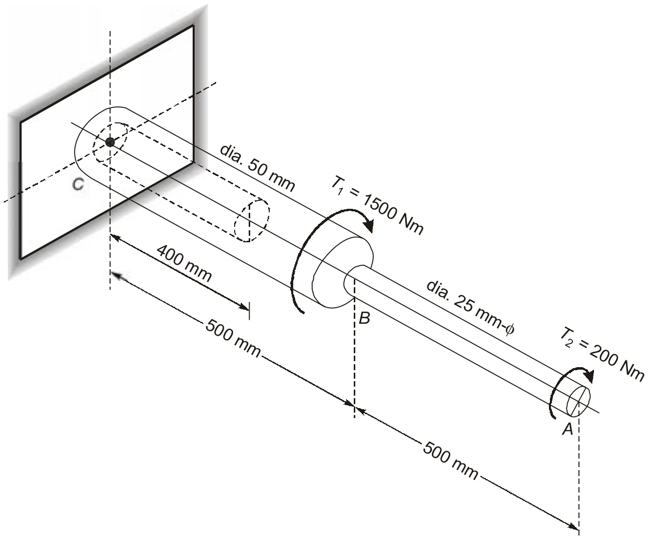
$$\theta = 0.0174 \text{ radian} = 0.997^\circ$$

4.4 A circular aluminium shaft of dimensions shown in the figure is subjected to two torques, T_1 = 1500 Nm and T_2 = 200 Nm as shown. The left end of the shaft is completely fixed. Determine

(i) the angle of twist of the shaft at the right end,

(ii) the percentage change in the angle of twist of a concentric hole of diameter 40 mm and length 400 mm is drilled through the shaft, starting from the fixed end (indicated by dotted lines into the fig.)

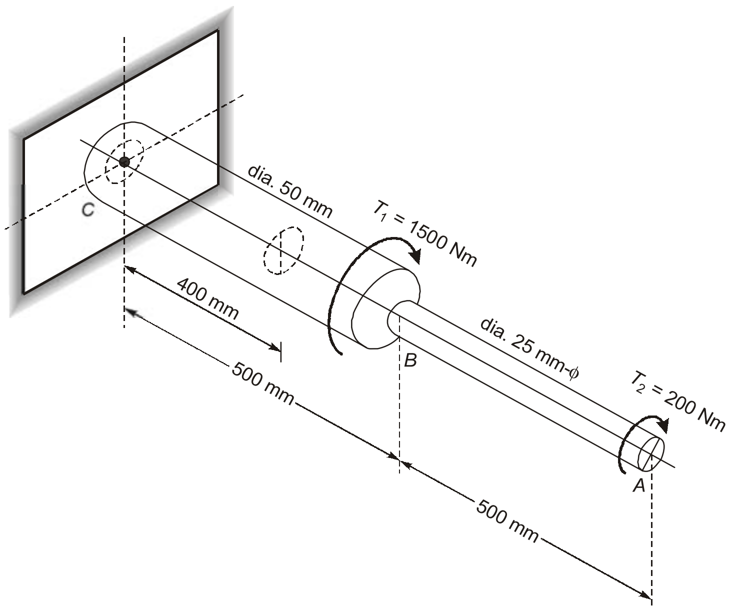
The shear modulus of aluminium can be taken as 70 × 10⁹ N/m².



[15 marks : 2016]

Solution:

(i)



Angle of twist of right end A, $\theta = \theta_{AB} + \theta_{BC}$ (shaft in series)

$$T_{AB} = T_2 = 200 \text{ Nm}$$

$$T_{BC} = T_1 + T_2 = (1500 + 200) = 1700 \text{ Nm}$$

Also, $G_{At} = 70 \times 10^2 \text{ N/m}^2$

We know, $$\theta = \frac{Tl}{GJ}$$

∴ $$\theta_i = \theta_{AB} + \theta_{BC}$$

$$= \frac{T_{AB} l_{AB}}{GJ_{AB}} + \frac{T_{BC} l_{BC}}{GJ_{BC}}$$

$$J_{AC} = \frac{\pi d_1^4}{32} = \frac{\pi \times (25)^4}{32} = 38349.52 \text{ mm}^4$$

$$J_{BC} = \frac{\pi d_2^4}{32} = \frac{\pi \times (50)^4}{32} = 613592.32 \text{ mm}^4$$

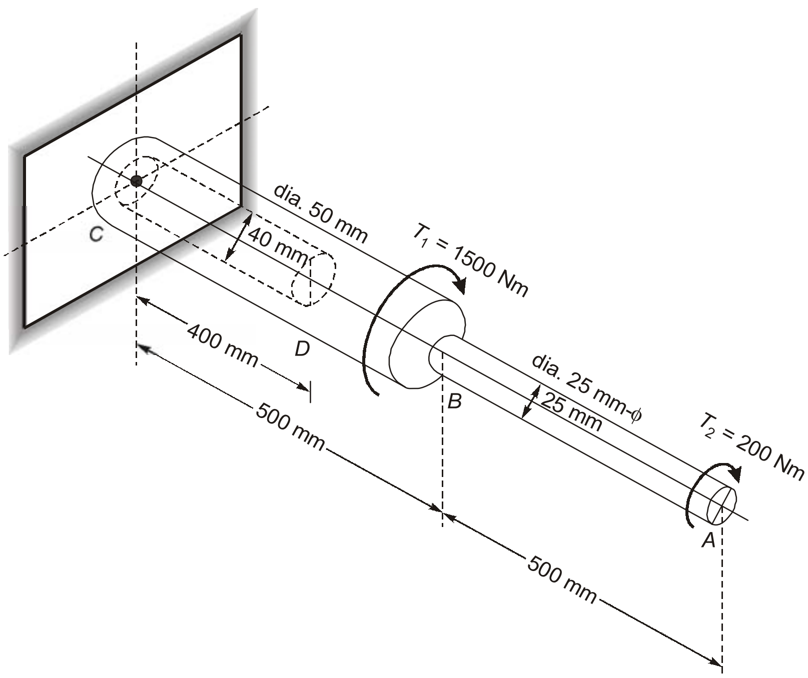
$$\Rightarrow \quad \theta_{AB} = \frac{200 \times 10^3 \times 500 \times 10^6}{70 \times 10^9 \times 38349.52} = 0.0373 \text{ radian}$$

and

$$\theta_{BC} = \frac{1700 \times 10^3 \times 500 \times 10^6}{70 \times 10^9 \times 613592.32} = 0.0198 \text{ radian}$$

$$\theta_i = 0.0373 + 0.0198 = 0.0571 \text{ radians}$$

(ii)



$$J_{BD} = J_{BC} \text{ (Calculated previously)}$$

$$J_{DC} = \frac{\pi}{32}\left(50^4 - 40^4\right) = 362264.91 \text{ mm}^4$$

$$\theta_f = \theta_{AB} + \theta_{BD} + \theta_{CD}$$

$$= 0.0373 + \frac{T_{BD} \times 100 \times 10^6 \times 10^3}{613592.32 \times 79 \times 10^9} + \frac{T_{BD} \times 400 \times 10^6 \times 10^3}{362264.91 \times 79 \times 10^9}$$

Put $T_{BD} = 1700$ Nm

$$\Rightarrow \quad 0.0373 + 0.0035 + 0.0237 = 0.0645 \text{ radians}$$

$$\% \text{ increase in } \theta = \left(\frac{0.0645 - 0.0565}{0.0565}\right) \times 100 = 14.26\%$$

4.5 A solid cylindrical shaft is to transmit 300 kW power at 100 rpm. If the shear stress is not to exceed 80 N/mm^2, find its diameter. What percent saving in weight would be obtained if this shaft is replaced by a hollow one whose internal diameter equals to 0.6 of external diameter, the length, the material and maximum shear stress being the same?

[15 Marks : 2023]